BT-3/D-23 **43142**

MATHEMATICS-III

BS-205A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Section. All questions carry equal marks.

Section A

1. Examine the convergence of the series :

(i) $\dfrac{1}{2\sqrt{1}}+\dfrac{x^2}{3\sqrt{2}}+\dfrac{x^4}{4\sqrt{3}}+\dfrac{x^6}{5\sqrt{4}}+\ldots\infty$

(ii) $\dfrac{1}{2}+\dfrac{2}{3}x+\left(\dfrac{3}{4}\right)^2x^2+\left(\dfrac{4}{5}\right)^3x^3+\ldots\infty$

2. Prove that $x^2=\dfrac{\pi^2}{3}+4\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{\cos nx}{n^2}$, $-\pi<x<\pi$.

Hence show that :

$$\sum\frac{1}{n^2}=\frac{\pi^2}{6}.$$

Section B

3. Solve :

$$\frac{2x}{y^3}dx+\frac{(y^2-3x^2)}{y^4}dy=0.$$

4. Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$ using the method variation of parameter for finding the particular integral.

Section C

5. Change the order of integration in $I=\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$.

6. Evaluate $\iint_D (x+2y)dx\,dy$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

Section D

7. For the function $\phi(x,y)=\frac{x}{x^2+y^2}$, find the magnitude of the directional derivative along a line making an angle 30 with the positive x-axis at (0, 2).

8. State Green's Theorem for a plane and verify the same for $\int_C (3x^2-8y^2)dx+(4y-6xy)dy$, where C is the boundary of the region bounded by $x \geq 0$, $y \leq 0$ and $2x - 3y = 6$.